Mathematics
Higher Level
Paper 1

Name

Date: ____________________

2 hours

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

exam: 12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A (56 marks)

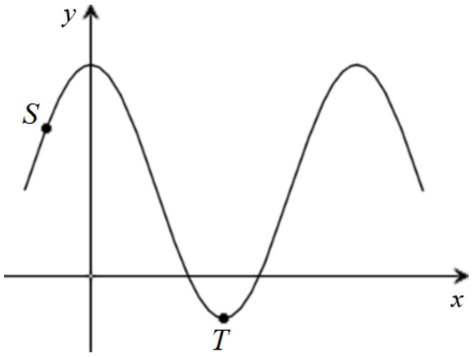
Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Given that $px^3+qx^2-9x+18$ is exactly divisible by $(x+2)(x-3)$, find the value of p and the value of q.

2. [Maximum mark: 6]

The diagram below shows a curve with equation $y = 2 + k\cos x$, defined for $-\frac{\pi}{2} \le x \le \frac{5\pi}{2}$.



The point S lies on the curve and has coordinates $\left(-\frac{\pi}{3}, \frac{7}{2}\right)$. The point T with coordinates (a, b) is the minimum point.

(a) Show that $k = 3$. [2]

(b) Hence, find the value of a and the value of b. [4]

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3. [Maximum mark: 6]

A geometric series has a positive common ratio r. The series has a sum to infinity of 9 and the sum of the first two terms is 5. Find the first three terms of the series.

4. [Maximum mark: 5]

The probability density function for a random variable X is given by

$$f(x)=\begin{cases} cx\mathrm{e}^{-x}, & 0\le x\le 1 \\ 0, & \text{otherwise} \end{cases}$$ where c is a real number.

Find the value of c.

5. [Maximum mark: 7]

Consider the equation $ax^2 + 7x - 2a = 0$ that has two distinct solutions for x.

(a) Given that $x = -3$ is a solution of $ax^2 + 7x - 2a = 0$, find the value of a and the other solution for x. [4]

(b) Hence, express $\dfrac{5x-7}{ax^2+7x-2a}$ as the sum of two fractions. [3]

6. [Maximum mark: 7]

A curve has equation $4xy - y^2 - x^3 = 0$ for $x > 0,\ y > 0$. The graph of the curve has a vertical tangent at point R. Find the coordinates of R.

7. [Maximum mark: 7]

Prove by mathematical induction that $2^n > 2n+1$ for all $n \geq 3, n \in \mathbb{Z}$.

8. [Maximum mark: 7]

Solve for x in each of the following equations:

(a) $\log_2\left(5x^2 - x - 2\right) = 2 + 2\log_2 x$. [3]

(b) $3^{x+1} = 2^{2-x}$. Express the answer in the form $\frac{\ln a}{\ln b}$, $a, b \in \mathbb{Q}$. [4]

9. [Maximum mark: 6]

The coefficients of x^2 in the expansions $(1+x)^{2n}$ and $(1+15x^2)^n$ are equal. Given that n is a positive integer, find the value of n.

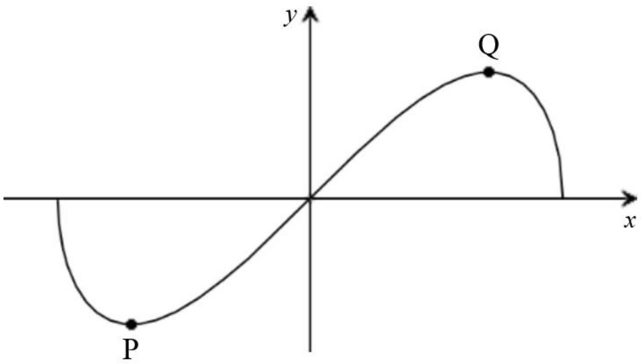
Do **not** write solutions on this page.

Section B (54 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 24]

The diagram shows the graph of the function defined by $f(x) = x\sqrt{1-x^2},\ -1 \le x \le 1$.



The function has a minimum at the point P and a maximum at point Q.

(a) Show that $f'(x) = \dfrac{1-2x^2}{\sqrt{1-x^2}}$. [4]

(b) Find the coordinates of P, and the coordinates of Q. [4]

(c) Find the total area enclosed by the graph of f and the x-axis. [5]

(d) The graph of f is rotated 2π radians about the x-axis, forming a solid.

Show that the total volume of this solid is $\dfrac{4\pi}{15}$. [5]

The function g is defined as $g(x) = 2f(x-3)$.

(e) Determine the domain and the range of g. [4]

(f) Another solid is formed when the graph of g is rotated 2π radians about the x-axis. Write down the total volume of this solid. [2]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

Consider the points $A(8, -4, 5)$, $B(5, -3, 4)$ and $C(3, -2, 5)$.

(a) Find the vector $\overrightarrow{AC} \times \overrightarrow{AB}$. [4]

(b) Determine the area of triangle ABC. [3]

(c) Plane Π_1 contains triangle ABC. Show that a Cartesian equation for Π_1 is $2x + 5y - z = -9$ [3]

A second plane Π_2 is defined by the Cartesian equation $\Pi_2 : x + by + cz = -6$, where b and c are constants. Plane Π_2 is perpendicular to plane Π_1 and the two planes intersect at a line with the Cartesian equation $\frac{x+1}{-16} = \frac{y+1}{5} = \frac{z-2}{-7}$.

(d) Find the value of b, and the value of c. [4]

A third plane, Π_3, is defined by the Cartesian equation $\Pi_3 : x + 2y - 2z = 9$.

(e) Given that Π_1, Π_2 and Π_3 intersect at point P, find the coordinates of P. [3]

12. [Maximum mark: 13]

Consider the complex number $w = \cos\theta + i\sin\theta$.

(a) Show that $w^n + \frac{1}{w^n} = 2\cos n\theta$ where $n \in \mathbb{Z}^+$. [3]

(b) Hence, write down an expression, in terms of $\cos\theta$, for $\left(w + \frac{1}{w}\right)^5$. [1]

(c) Show that $\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$. [4]

(d) Hence, find all the solutions of $\cos 5\theta + 5\cos 3\theta + 12\cos\theta = 0$ in the interval $0 \le \theta < 2\pi$. [5]